Forces and Motion - Questions

June 2017 Mathematics Advanced Paper 1: Mechanics 1

1.

1. Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q.

(6)

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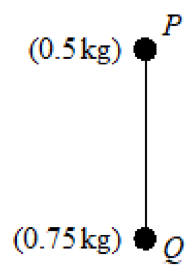

Figure 2

A vertical light rod PQ has a particle of mass 0.5 kg attached to it at P and a particle of mass 0.75 kg attached to it at Q, to form a system, as shown in Figure 2. The system is accelerated vertically upwards by a vertical force of magnitude 15 N applied to the particle at Q. Find the thrust in the rod.

(6)

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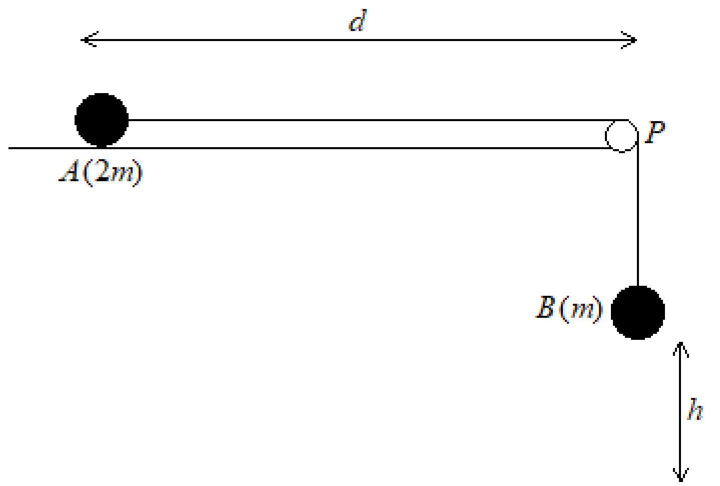

Figure 3

Two particles, A and B, have masses $2m$ and m respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a fixed rough horizontal table at a distance d from a small smooth light pulley which is fixed at the edge of the table at the point P. The coefficient of friction between A and the table is μ, where $\mu < \frac{1}{2}$.

The string is parallel to the table from A to P and passes over the pulley. Particle B hangs freely at rest vertically below P with the string taut and at a height h, $(h < d)$, above a horizontal floor, as shown in Figure 3. Particle A is released from rest with the string taut and slides along the table.

(*a*) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(*b*) Hence show that, until B hits the floor, the acceleration of A is $\frac{g}{3}(1 - 2\mu)$.

(3)

(*c*) Find, in terms of g, h and μ, the speed of A at the instant when B hits the floor.

(2)

After B hits the floor, A continues to slide along the table. Given that $\mu = \frac{1}{3}$ and that A comes to rest at P,

(*d*) find d in terms of h.

(5)

(*e*) Describe what would happen if $\mu = \frac{1}{2}$

(1)

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4.

7. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle *P*.

The force $\mathbf{F}_1$ is given by $\mathbf{F}_1 = (-\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + \mathbf{j})$.

Given that the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + 3\mathbf{j})$,

(*a*) find $\mathbf{F}_2$.

(7)

The acceleration of *P* is $(3\mathbf{i} + 9\mathbf{j})$ m s^{-2}. At time $t = 0$, the velocity of *P* is $(3\mathbf{i} - 22\mathbf{j})$ m s^{-1}.

(*b*) Find the speed of *P* when $t = 3$ seconds.

(4)

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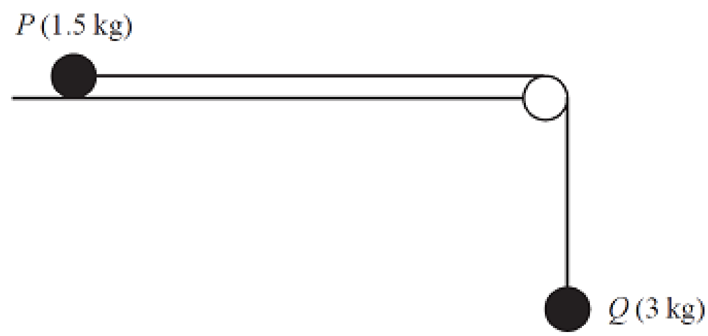

Figure 3

Two particles *P* and *Q* have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle *P* is held at rest on a fixed rough horizontal table. The coefficient of friction between *P* and the table is $\frac{1}{5}$. The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle *Q* hangs freely at rest vertically below the pulley, as shown in Figure 3. Particle *P* is released from rest with the string taut and slides along the table.

Assuming that *P* has not reached the pulley, find

(*a*) the tension in the string during the motion,

(8)

(*b*) the magnitude and direction of the resultant force exerted on the pulley by the string.

(4)

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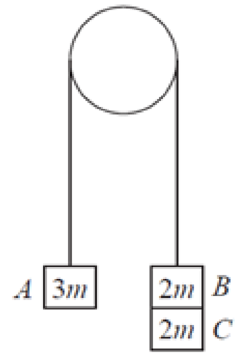

Figure 5

Three particles A, B and C have masses $3m$, $2m$ and $2m$ respectively. Particle C is attached to particle B. Particles A and B are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5. The system is released from rest and A moves upwards.

(*a*) (i) Show that the acceleration of A is $\frac{g}{7}$.

(ii) Find the tension in the string as A ascends.

(7)

At the instant when A is 0.7 m above its original position, C separates from B and falls away. In the subsequent motion, A does not reach the pulley.

(*b*) Find the speed of A at the instant when it is 0.7 m above its original position.

(2)

(*c*) Find the acceleration of A at the instant after C separates from B.

(4)

(*d*) Find the greatest height reached by A above its original position.

(3)

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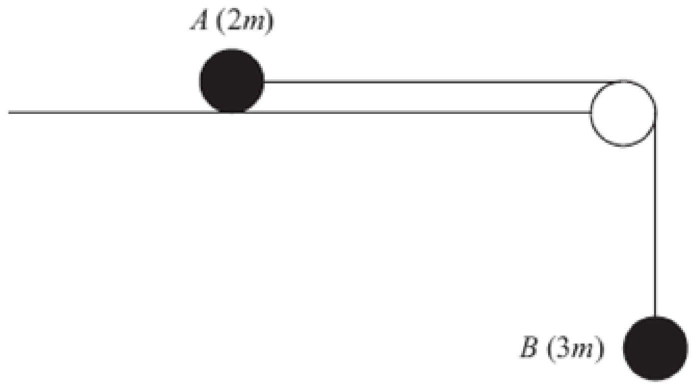

Figure 2

Two particles A and B have masses $2m$ and $3m$ respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs at rest vertically below the pulley with the string taut, as shown in Figure 2. Particle A is released from rest. Assuming that A has not reached the pulley, find

(a) the acceleration of B,

(5)

(b) the tension in the string,

(1)

(c) the magnitude and direction of the force exerted on the pulley by the string.

(4)

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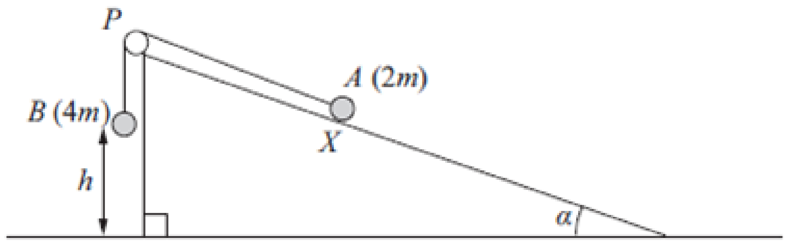

Figure 5

Figure 5 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(*a*) give a reason why the magnitudes of the accelerations of the two particles are the same,

(1)

(*b*) write down an equation of motion for each particle,

(4)

(*c*) find the acceleration of each particle.

(5)

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

(*d*) find the distance XY in terms of h.

(6)

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Figure 3

Two particles P and Q, of mass 0.3 kg and 0.5 kg respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane.

At time $t = 0$, a constant force **F** of magnitude 4 N is applied to Q in the direction PQ, as shown in Figure 3. The system moves under the action of this force until $t = 6$ s. During the motion, the resistance to the motion of P has constant magnitude 1 N and the resistance to the motion of Q has constant magnitude 2 N.

Find

(*a*) the acceleration of the particles as the system moves under the action of **F**,

(3)

(*b*) the speed of the particles at $t = 6$ s,

(2)

(*c*) the tension in the rod as the system moves under the action of **F**.

(3)

At $t = 6$ s, **F** is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

(*d*) the distance moved by P as the system decelerates,

(4)

(*e*) the thrust in the rod as the system decelerates.

(3)

10.

3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(*a*) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Find

(*b*) the magnitude of $\mathbf{R}$,

(2)

(*c*) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

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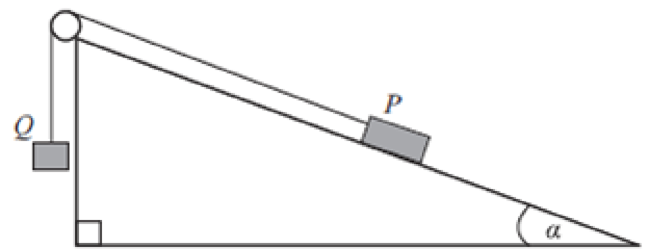

Figure 2

Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2}.

Find

(a) the magnitude of the normal reaction of the inclined plane on P,

(2)

(b) the value of m.

(8)

When the particles have been moving for 0.5 s, the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest.

(6)

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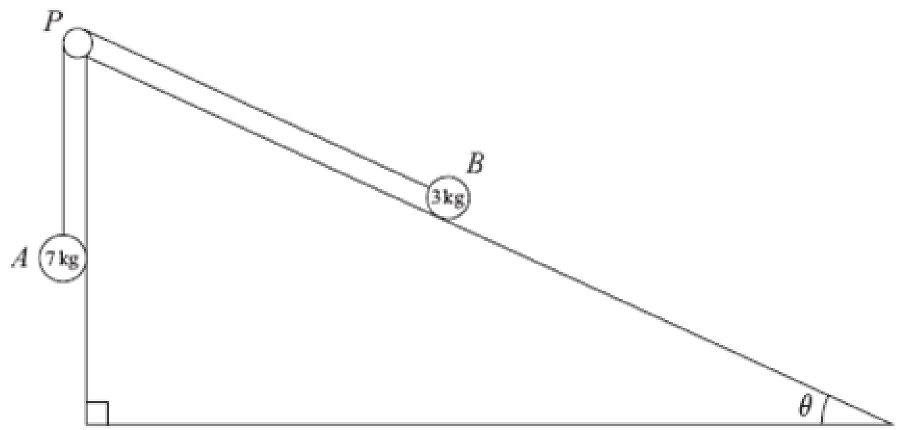

Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(*a*) Find the magnitude of the acceleration of B immediately after release.

(10)

(*b*) Find the speed of B when it has moved 1 m up the plane.

(2)

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(*c*) find the time between the instants when the string breaks and when B comes to instantaneous rest.

(4)

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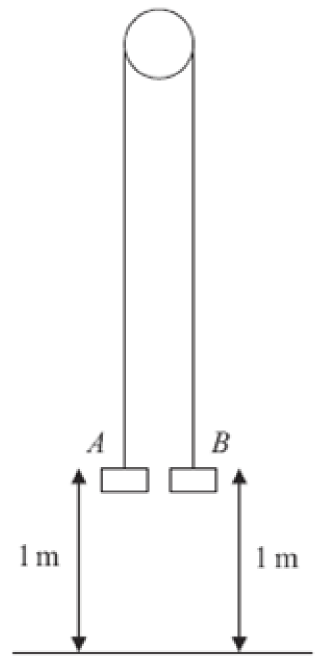

Figure 3

Two particles A and B have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in Figure 3. The particles are released from rest and in the subsequent motion B does not reach the pulley.

(*a*) Find the tension in the string immediately after the particles are released.

(6)

(*b*) Find the acceleration of A immediately after the particles are released.

(2)

When the particles have been moving for 0.5 s, the string breaks.

(*c*) Find the further time that elapses until B hits the floor.

(9)

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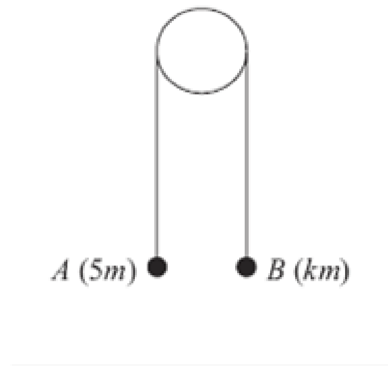

Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(*a*) Show that the tension in the string as A descends is $\frac{15}{4}mg$.

(3)

(*b*) Find the value of k.

(3)

(*c*) State how you have used the information that the pulley is smooth.

(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(*d*) Find the greatest height reached by B above the plane.

(7)